

A. ARITHMETIC (LINEAR) SEQUENCES

You have already dealt with linear sequences in Grade 10. In a linear sequence there is a constant difference between successive terms. In Grade 11 a formal equation to solving the n^{th} term of a linear sequence will now be introduced.

It is important to understand the notation used in number patterns.

The **first term** of a sequence is denoted as T_1 , the **second** as T_2 , the **third** term as T_3 and so on. The **last term** (or n^{th} term) of a sequence is denoted as T_n .

If a sequence is arithmetic, the difference between any two consecutive terms in the sequence will be the same. In other words: $T_2 - T_1 = T_3 - T_2 = T_4 - T_3 = \dots = T_n - T_{n-1}$

Derivation of the Arithmetic Sequence Formula

Let the first term of an arithmetic sequence equal a and the difference between successive terms equal d .

$$\therefore T_1 = a$$

$$T_2 = a + 1d \rightarrow 1 = 2 - 1$$

$$T_3 = a + 2d \rightarrow 2 = 3 - 1$$

$$\therefore T_n = a + (n - 1)d$$

Arithmetic Sequence Formula

$$T_n = a + (n - 1)d$$

Where: $a \rightarrow$ The first term of a sequence

$d \rightarrow$ The difference between two successive terms ($T_2 - T_1, T_3 - T_2, \dots$)

$n \rightarrow$ The position of the n^{th} term in the sequence (ie: Term 1, Term 2, ...)

Example 1: Given the sequence 12; 6; 0; -6; ...

a) Determine a general term for the n^{th} term of the sequence.

b) Determine the 50th term of the sequence.

c) Which term of the sequence is equal to -198?

a) Ensure that the sequence is arithmetic: $T_2 - T_1 = T_3 - T_2$

$$T_2 - T_1 = 6 - 12 = -6 \quad \text{and} \quad T_3 - T_2 = 0 - 6 = -6 \rightarrow \text{sequence is linear}$$

$$d = -6$$

$$a = 12$$

$$T_n = a + (n - 1)d$$

$$\therefore T_n = 12 + (n - 1)(-6)$$

$$b) T_{50} = 12 + (50 - 1)(-6)$$

$$= -282$$

$$c) T_n = -198$$

$$\therefore -198 = 12 + (n - 1)(-6)$$

$$\therefore -210 = (n - 1)(-6)$$

$$\therefore n - 1 = 35$$

$$\therefore n = 36 \rightarrow \text{the } 36^{\text{th}} \text{ term is equal to } -198 \quad (T_{36} = -198)$$