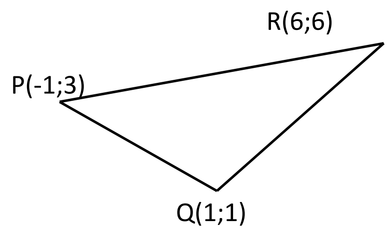
MATHEMATICS **ANALYTICAL GEOMETRY**

DISTANCE FORMULA: to find length or distance

A. TRIANGLES: Distance formula is used to show

- PERIMETER: sum of all the sides
- Scalene triangle: 3 unequal sides
- Isosceles triangle: 2 equal sides
- Equilateral triangle: 3 equal sides
- Right angled triangle:

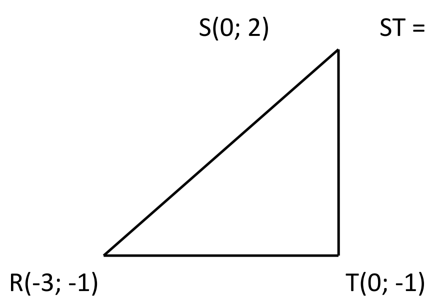
EXAMPLE 1 : Use the distance formula to show that the triangle below is right angled



PR = RQ= PQ=

Pythagorus:

EXAMPLE 2 : Use the distance formula to determine if the triangle below is right angled.



ST =

TR =

RS =

What other conclusion can you make about the triangle?

EXAMPLE 3 : The vertices of triangle UNR are given U(-6; 1) N(1; 4) R(-3; -6). Use the distance formula to determine the type of triangle. Is it also right angled?. Calculate the perimeter.

EXAMPLE 4: The vertices of triangle ABC are given, A(-8; 9) N(-2; -1) R(7; 3). Use the distance formula to determine the type of triangle. Calculate the perimeter.

EXAMPLE 5: The vertices of triangle PQR are given, P(3;) Q(0; 0) R(6; 0). Use the distance formula to determine the type of triangle. Calculate the perimeter.

EXAMPLE 6 : Triangle ABC is an isosceles triangle with vertices A(-7; -2) ; B(-1; Y) and C. AC = 6 and AB is equal to AC. Find the coordinate of y.

EXAMPLE 7: Triangle DEF is equilateral with vertices D(3; 0) E(-7; 0) F(-1; Y) . Find y.

B. QUADRILATERALS : Distance formula is used to show:

PARALLELOGRAM
RHOMBUS
RECTANGLE
SQUARE
KITE
TRAPEZIUM

1) PARALLELOGRAM **CHARACTERISTICS**

- opposite sides equal and parallel
- diagonals not equal
- diagonals are cut in half (BISECT) at the midpoint
- corner angles are NOT 90˚
- opposite angles are supplementary
- sum of 4 corner angles = 360˚

2) RHOMBUS

- 4 equal sides
- opposite sides equal and parallel
- diagonals not equal
- diagonals are cut in half (BISECT) at the midpoint at 90˚
- corner angles are NOT 90˚
- opposite angles are supplementary
- sum of 4 corner angles = 360˚

3) <u>RECTANGLE</u>

- opposite sides equal and parallel
- diagonals ARE EQUAL
- diagonals are cut in half (BISECT) at the midpoint
- corner angles ARE 90˚
- opposite angles are supplementary
- sum of 4 corner angles = 360˚

4) <u>SQUARE</u>

- 4 equal sides
- opposite sides equal and parallel
- diagonals ARE EQUAL
- diagonals are cut in half (BISECT) at the midpoint at 90˚
- corner angles are 90˚
- diagonals bisect corner angles into 45˚ + 45˚
- opposite angles are supplementary
- sum of 4 corner angles = 360˚

5) <u>KITE</u>

- ADJACENT sides are Equal
- diagonals are not equal
- the long diagonal bisects the short diagonal at its midpoint
- the long diagonal bisects the short diagonal at 90˚
- the long diagonal bisects its corner angles
- the angles at the ends of the short diagonal are equal
- sum of the corner angles = 360˚

6) <u>TRAPEZIUM</u>

- 4 sides which are not equal BUT
- ONE PAIR OF OPPOSITE SIDES ARE PARALLEL

NOTE: With the parallelogram, rectangle, rhombus and square, if you can show that **2 PAIRS OF OPPOSITE SIDES ARE EQUAL** then the opposite sides are also **PARALLEL.**

In each of the following questions below the four vertices of the quadrilateral are given. Draw a rough diagram and use the distance formula to determine the type of quadrilateral.

<u>EXAMPLE 1</u>: R(-1; 1) A(4; 2) C(2; -1) E(-3; -2)

EXAMPLE 2: M(0; 0) E(8; 0) T(8; 8) S(0; 8)

EXAMPLE 3 : W(1; 1) X(5; 3) Y(7; 7) Z(3; 5)

EXAMPLE 4: A(-2; 2) B(4; 2) C(4; 8) D(-2; 8)

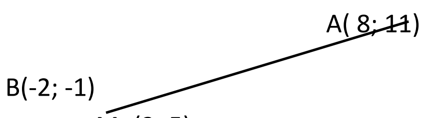
EXAMPLE 5: D(-5; 2) E(3; 0) F(8; 8) G(-4; 5)

MIDPOINT FORMULA

Use the midpoint to show :

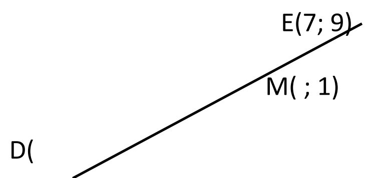
- -the midpoint of a line
- -to find the coordinates of another point
- -diagonals of parallelogram, rectangle, square, rhombus bisect at the midpoint

EXAMPLE 1: Find the midpoint of line AB



M=(3; 5)

EXAMPLE 2: Use the midpoint to find the coordinates of point D



1 =

Fraction and equals means CROSS MULTIPLY

2x1= 4(2x1 =

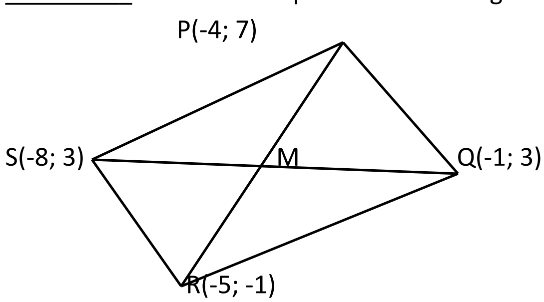
2 = 4 2 = 2 – 28 = 4

2 -9 = -26 = 4 -7

=

-6.5 = Therefore D(-6.5; -7)

EXAMPLE 3 : Find the midpoint for both diagonals.

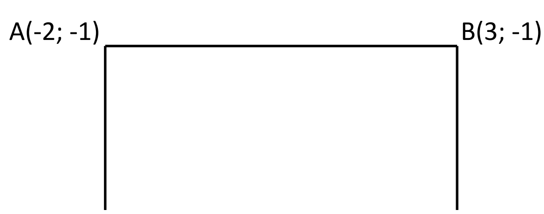


Find the length of both diagonals

Find the length of SM and QM

Do the diagonals bisect each other at the midpoint?

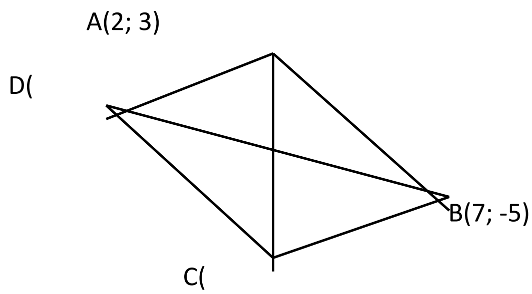
EXAMPLE 4



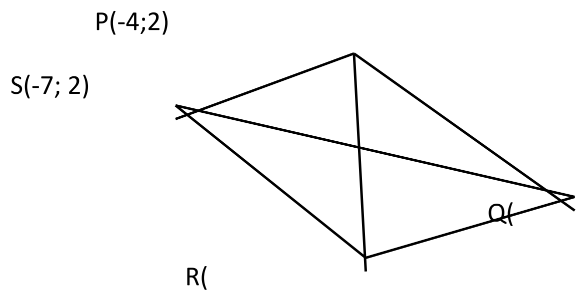
Show that the diagonals BISECT at the midpoint.

Show that ABCD is a rectangle.

EXAMPLE 5: ABCD is a parallelogram with midpoint (Find the coordinates of C and D



EXAMPLE 6: PQRS is a parallelogram with midpoint Find the coordinates of Q and R



COLLINEAR POINTS = Points on the same straight line

ie. SAME GRADIENT BETWEEN THE POINTS

(TO PROVE POINTS ARE COLLINEAR YOU MUST SHOW THAT THE GRADIENTS BETWEEN POINTS ARE EQUAL)

gradient = m =

EXAMPLE 1 : Prove points A(-3; 5), B(-1;1) C(2;-5) are collinear.

EXAMPLE 2: Prove points T(5; 4) N(1; 2) and R(-5;-1) are collinear.

EXAMPLE 3: Points V(-6; 4) W(2; -2) and P(10; -y) are collinear. Find y.

EXAMPLE 4: Points L(1; -8) R(x; -17) S(2;1) are collinear. Find x.

EQUATION OF A STRAIGHT LINE

y = mx + c

m =

If m is positive the graph goes to the right ↗

If m is negative the graph goes to the left ↖

<u>y- intercept</u>

Point (o;y) where line crosses y-axis

<u>How to calculate:</u>

1) let x = 0

2) substitute m and a point (x;y) into equation y = mx + c to get c.

NOTE: X-INYERCEPT : Where the graph cuts the x-axis: let y = 0

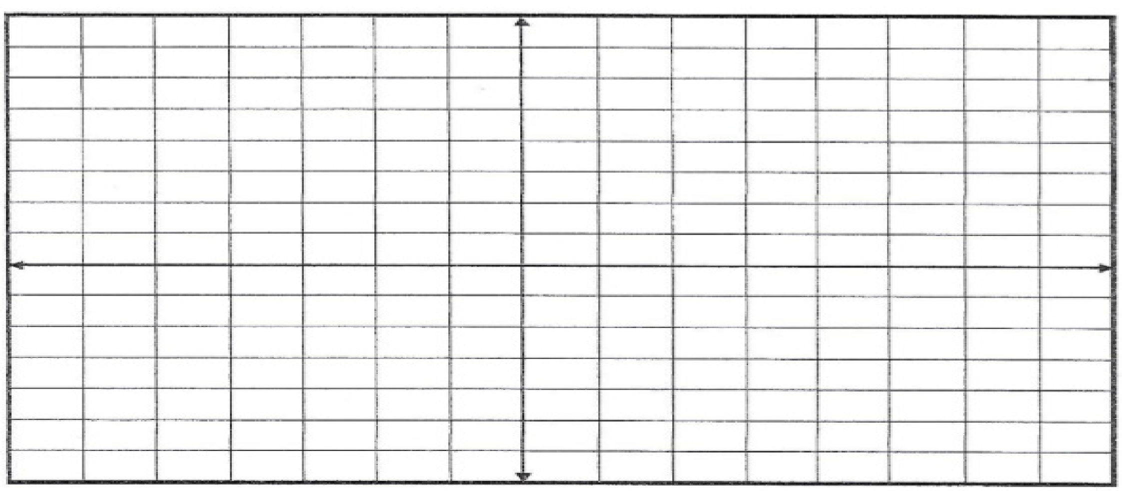
<u>A.DRAWING THE STAIGHT LINE</u>

Only need two points: 1) y-intercept (0;y) : let x = 0
2) x-intercept (x; 0): let y = 0

<u>Example 1:</u> Draw the graphs of y= 2x + 8 and y = -4x + 2 on the same axes.

y = 2x + 8
<u>y-intercept</u>: let x = 0
y = 2.0 + 8 = 8 ie. **(0; 8)**
x-intercept: let y = 0
0 = 2x + 8
-2x = 8 Therefore x = -4 **(-4; 0)**

y = -4x + 2
<u>y-intercept:</u> let x = 0
y = -4.0 + 2 = 2 ie. **(0;2)**
x-intercept: let y =0
0 = -4x + 2
4x = 2 Therefore x = **(; 0)**

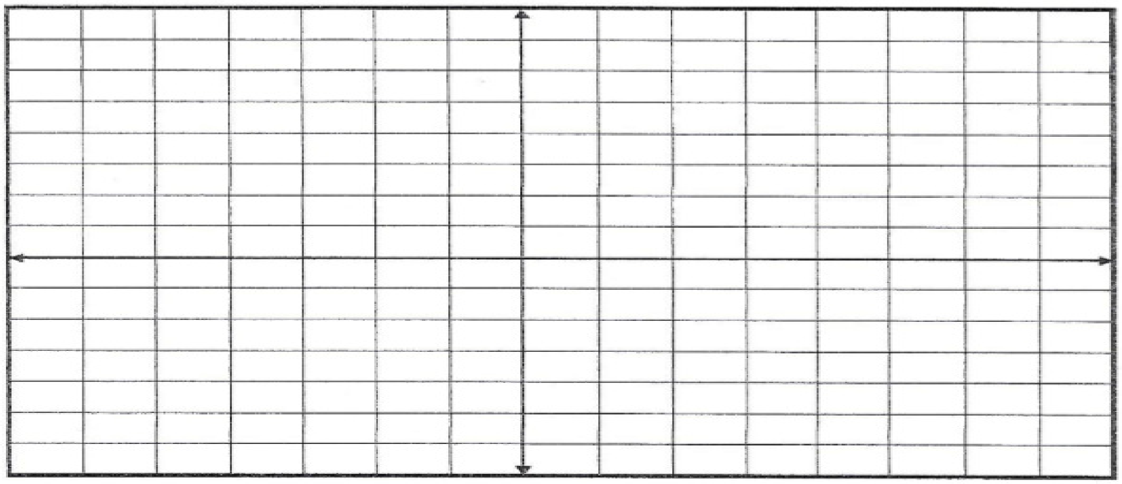


<u>Find the point of intersection:</u> make the 2 equations equal to each other

-4x + 2 = 2x +8
-4x -2x = 8 -2
-6x = 6 x = -1

substitute x = into y = 2x + 6
y = 2.+ 6 = 4
ie. **(is point intersection**

<u>Example 2</u>: Draw y = -3x -12 and y = 2x -2 and find the point of intersection.

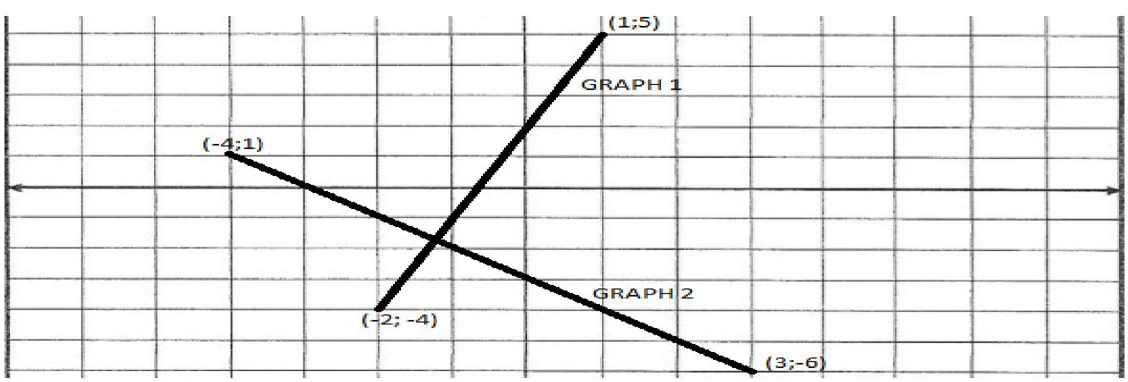


Point of intersection:

Finding the Equation of a Straight Line

Method:
- Write down y = mx + c
- calculate gradient
- substitute m into y = mx + c
- find c by substituting an (x;y) from a point on the line into y = mx + c
- write equation y = mx + c

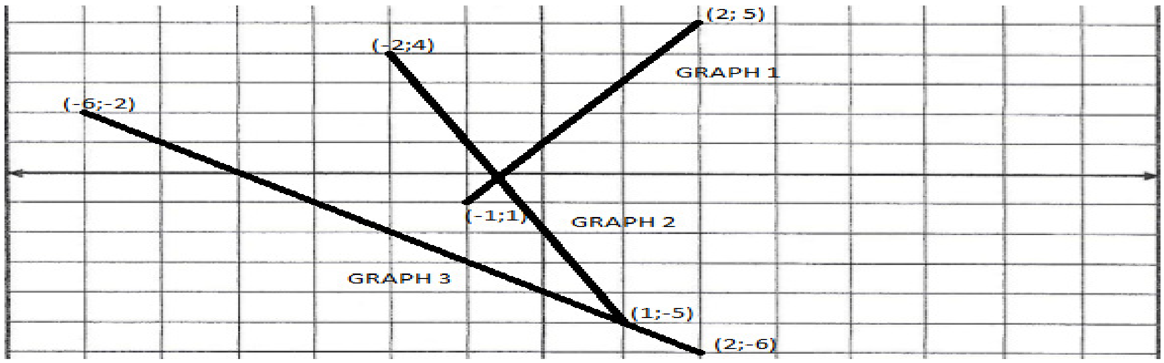
Example 1: Find the equation of graph 1 and graph 2.



Graph 1

Graph 2

Example 2: Find the equations of graphs 1, 2 and 3.



Graph 1 Graph 2 Graph 3

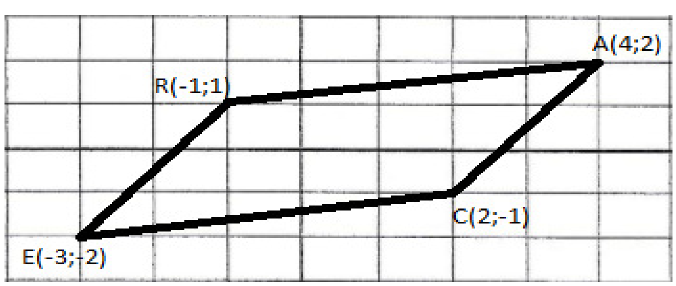
PARALLEL LINES

Parallel lines have the **SAME gradient.**

Method: Calculate = and = If the lines are PARALLEL.

Example1:

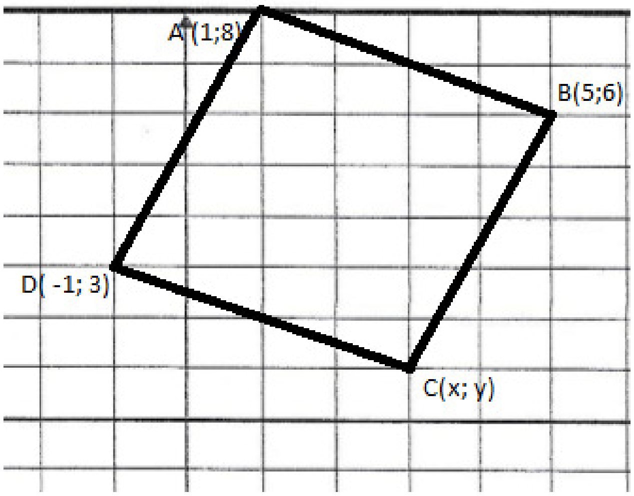
1.1 Show that AR and RE



1.2 Hence show that ACER is a parallelogram

Example 2: Given quadrilateral ABCD.

2.1 Find the midpoint of BD.

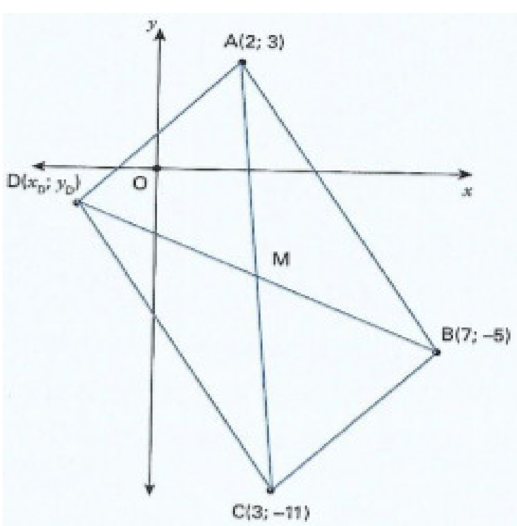


2.2 Determine the coordinates of C(x;y)

2.3 Show that ABCD is a parallelogram.

<u>Example 3</u> Given quadrilateral ABCD.

3.1 Find the midpoint of AC.

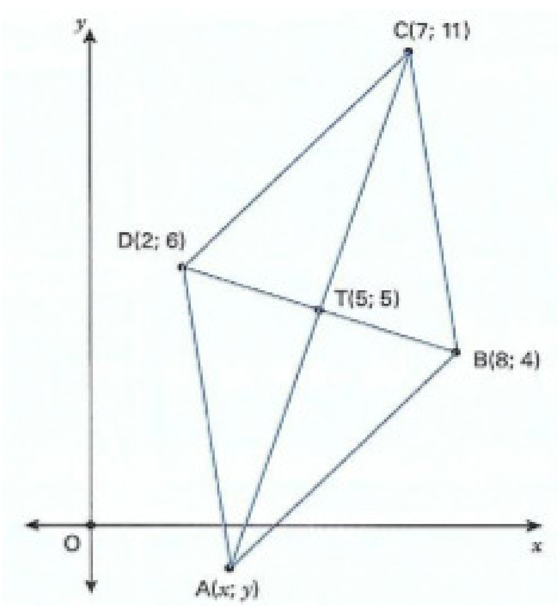


3.2 Find the coordinates of D(x;y)

3.3 Show that ABCD is a parallelogram

Example 4. Given quadrilateral ABCD.

4.1 Calculate the coordinates of point A



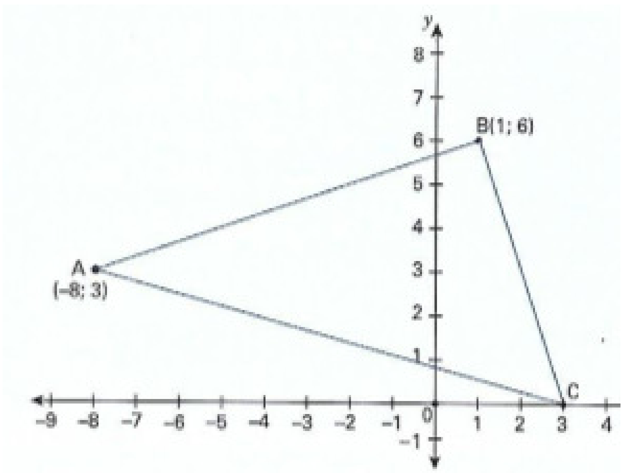
4.2 Show that ABCD is a Rhombus.

PERPENDICULAR LINES

Method: Calculate = and = If the lines are **Perpendicular**

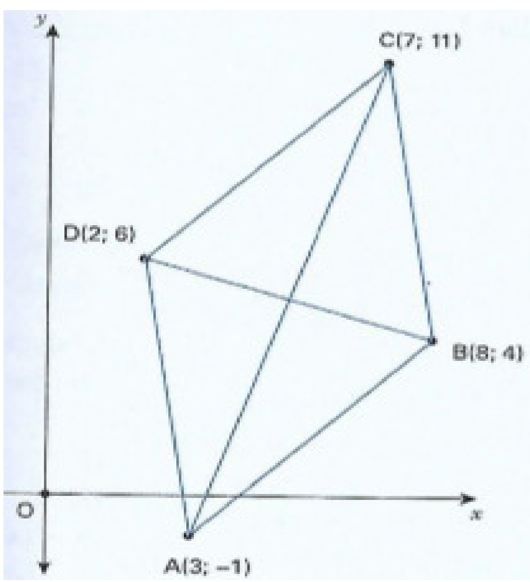
Example 1:

1.1 Show that triangle ABC is a right angled triangle.



Example 2:

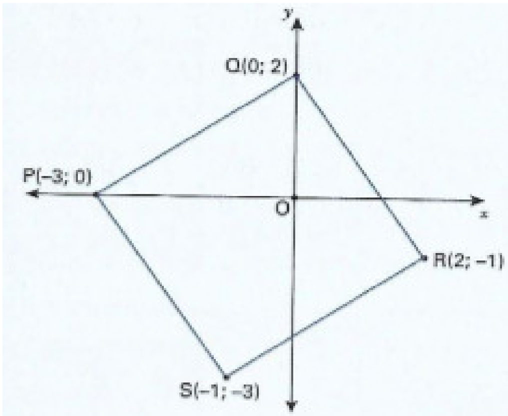
2.1 Show that AB is parallel to DC



2.2 Show that AC is perpendicular to DB.

Example 3: Quadrilateral PQRS

3.1 Determine the coordinates of M, the midpoint of PR

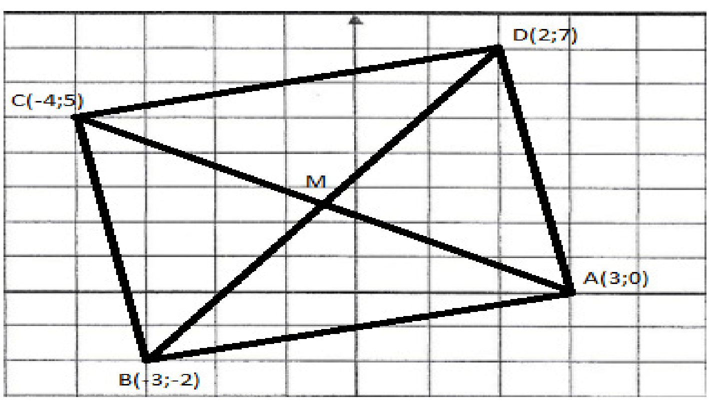


3.2 Show that PR and QS bisect each other (cut each other in half at the same midpoint)

3.3 Show that angle PQR is 90.

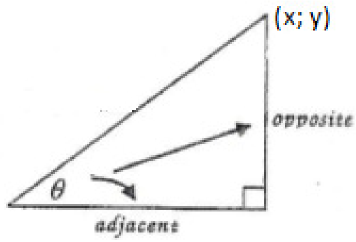
<u>Example 4</u>

4.1 Show that the diagonals bisect at the midpoint M



4.2 Prove that ABCD is a rectangle

<u>ANGLE OF INCLINATION</u> = <u>the angle between a straight line and the x-axis.</u>

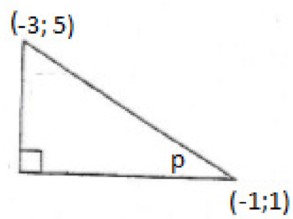


tan =

tan = (*gradient of line*)

=

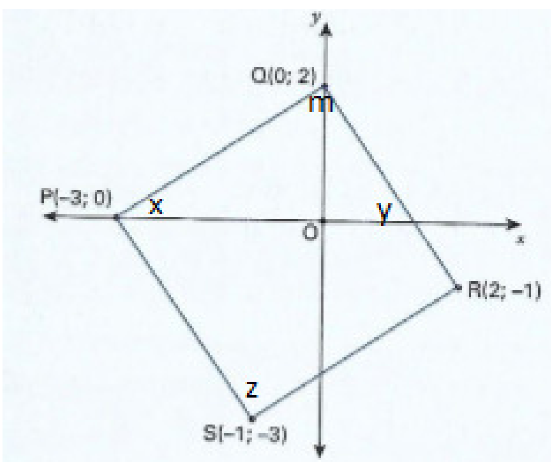
If the graph has a **negative gradient**, leave off the minus and calculate as normal.



=

p = = = p = Now leave off the minus and calculate. p = = 63.43

Example 1: 1.1 Calculate the size of angel x and y.



= = x = == y = =

x = y = Abandon the minus x = 33,69

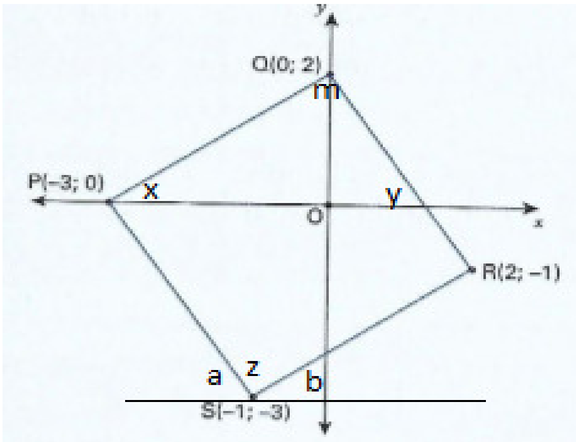
y = = 56,31

Therefore angle m = 180 - 56,31 - 33,69 = 90

1.1 Calculate the size of angle Z

-Angle Z is not on the x-axis

-Therefore we need to create an x-axis at S by drawing a horizontal line through S

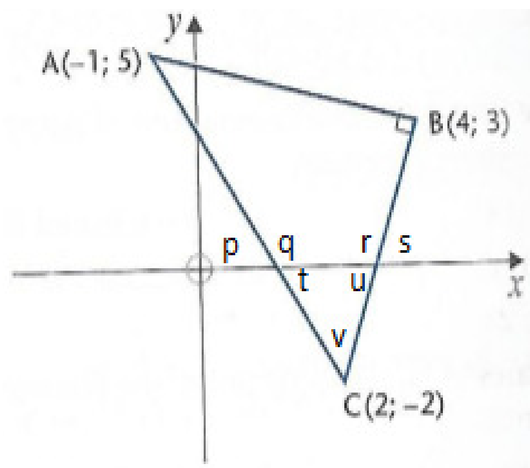


a = Abandon the minus b =

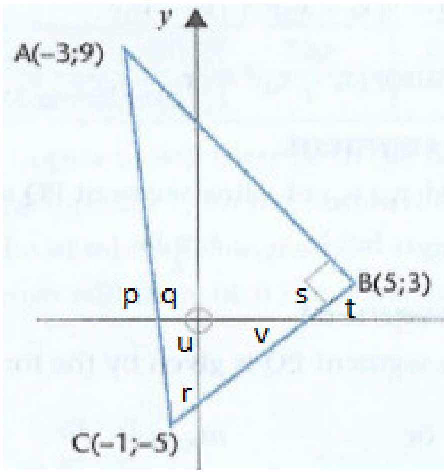
a = = 56,31 b = 33,69

Therefore Z = 180 - 56,31 - 33,69 = 90

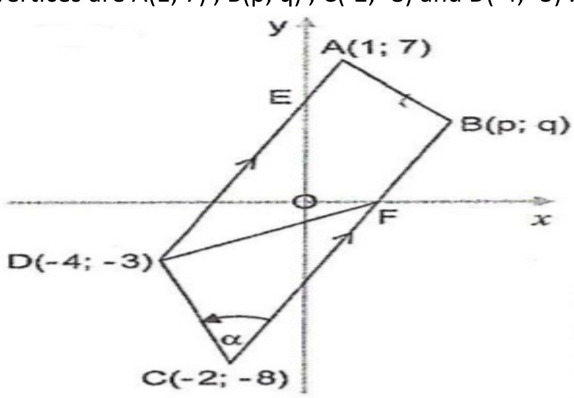
<u>Example 2</u>: Calculate the size of angles p, q, r, s, t, u, v.



<u>Example 3:</u> Calculate the size of angles p, q, s, t, u, v, r.



Example 4 In the diagram below, trapezium ABCD with ADBC is drawn. The coordinates of the vertices are A(1; 7) , B(p; q) , C(-2; -8) and D(-4; -3) . BC intersects the x-axis at F. DCB =



4.1 Calculate the gradient of AD. (3)

4.2 Determine the equation of BC in the form of $y = mx + c$. (3)

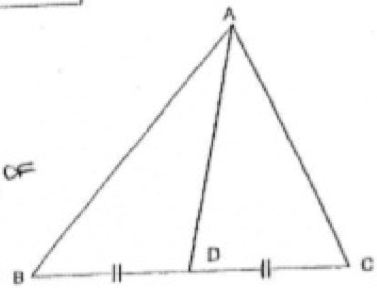
4.3 Determine the coordinates of point F. (2)

4.4 Show that (4)

[12]

EQUATION OF MEDIAN, ALTITUDE AND PERPENDICULAR BISECTOR

5. **Determining the equation of the median :**



AD is median

A **median** runs from a vertex of the triangle to the midpoint of the opposite side.

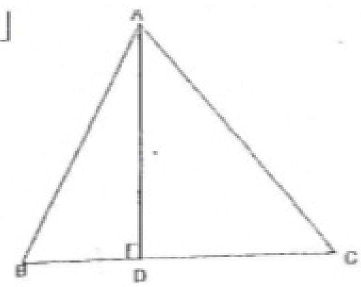
FIRST: DETERMINE MIDPOINT OF OPPOSITE SIDE

(2) USE POINT A (xy) and THE MIDPOINT (xy) TO FIND THE GRADIENT OF AD (the median)

(3) SUBSTITUTE A POINT (A or MIDPOINT) INTO $y = mx + c$ to get C

(4) WRITE DOWN EQUATION OF AD

6. **Determining the equation of the altitude :**



AD is altitude

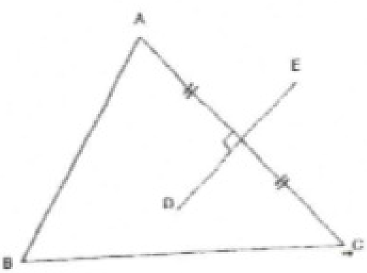
An **altitude** runs from a vertex of the triangle and is perpendicular to the opposite side.

FIRST: DETERMINE GRADIENT OF OPPOSITE SIDE BC. $\left(m = \frac{y_2 - y_1}{x_2 - x_1}\right)$

(2) USE $M_1 \times M_2 = -1$ to FIND THE GRADIENT OF AD, the ALTITUDE

(3) NOW SUBSTITUTE POINT A into $y = mx + c$ to GET C.

7. **Determining the equation of the perpendicular bisector :**



DE is perpendicular bisector

A **Perpendicular bisector** runs through the midpoint of a side and is perpendicular to that side. Usually it does not run through any of the vertices.

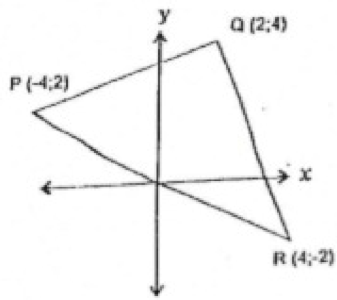
FIRST: DETERMINE THE MIDPOINT OF THE LINE AC AND GRADIENT OF AC

(2) USE $M_1 \times M_2 = -1$ to FIND GRADIENT OF PERPENDICULAR BISECTOR DE

(3) SUBSTITUTE MIDPOINT & GRADIENT DE INTO $y = mx + c$ to GET C.

EXERCISE

2\. In the diagram $P(-4;2), Q(2;4)$ and $R(4;-2)$ are the vertices of $\triangle PQR$.



.1 Prove that $\triangle PQR$ is a right-angled triangle.

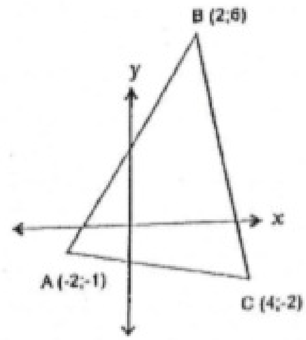
.2 Determine the lengths of PQ and QR.

.3 Hence, calculate the size of $\angle P$ without using a calculator.

.4 Determine the equation of the perpendicular bisector of QR.

.5 Is the perpendicular bisector parallel to PQ ? Give a reason for your answer.

3\. $A(-2;-1), B(2;6)$ and $C(4;-2)$ are the vertices of a triangle.



.1 Determine the coordinates of N, the midpoint of BC. Hence, determine the equation of the median AN.

.2 Determine the equation of the perpendicular bisector of AB.

.3 Determine the length of AC.

.4 If BA is produced to D so that A is the midpoint of BD, determine the coordinates of D.

4\. $A(-3;1), B(2;5), C(4;p)$ and $D(-1;-4)$ are the vertices of quadrilateral $ABCD$

4.1 Determine the value of p if:

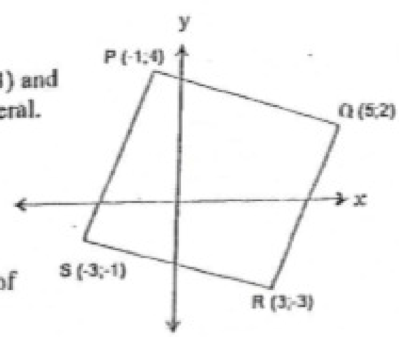
4.1.1 $AB \parallel DC$

4.1.2 $CB \perp AB$

4.1.3 $AD = BC$.

4.2 Determine the equation of AB and the coordinates of F, the *x-intercept* of AB.

5\. In the diagram $P(-1;4), Q(5;2), R(3;-3)$ and $S(-3;-1)$ are the vertices of a quadrilateral.



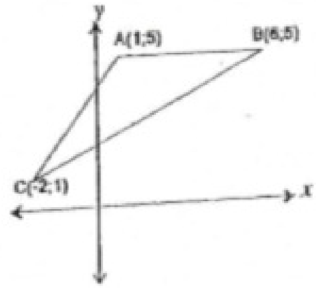
5.1 Prove that $PQRS$ is a parallelogram.

5.2 Determine the coordinates of the point of intersection of the diagonals of parallelogram $PQRS$.

5.3 Determine the equation of the perpendicular bisector of SR.

5.4 Calculate the size of $\angle SPQ$.

6. The coordinates of the vertices of $\triangle ABC$ are $A(1;5)$, $B(6;5)$ and $C(-2;1)$.



6.1 What is the equation of AB ?

6.2 What is the length of AB ?

6.3 Prove that $\triangle ABC$ is isosceles.

6.4 Determine the equation of the median AD, with D on BC.